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1992 J. Phys.: Condens. Matter 4 9309

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J. Phys.: Condens. Matter 4 (1992) 9309–9320. Printed in the UK

Non-equilibrium phase transitions in lattice systems with random-field competing kinetics: mean-field theory

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Received 5 May 1992, in final form 27 July 1992

Abstract. The study by kinetic mean-field techniques of a d-dimensional Ising system characterized by a sort of *dynamical disorder* reveals a rich phase diagram which exhibits a non-equilibrium tricritical point (only) for $d > 2$, and re-entrance phenomena. The system time evolution is stochastic due to the simultaneous action of several independent spin-flip mechanisms, each corresponding to a different applied magnetic field. Such competition brings about randomness and a type of frustration that may occur also in real systems. In fact, this models the actual case of a magnetic system under a very rapidly fluctuating field, for example. Furthermore, the system may be interpreted as a *non-equilibrium* random-field system which, unlike the familiar quenched and annealed cases, contains a fast random diffusion of disorder.

1. Introduction

Mathematically well defined systems with non-Hamiltonian constraints which prevent the realization of thermodynamic equilibrium allow the study of non-equilibrium steady states and phase transitions, which is an active area nowadays. Moreover, they are sometimes good models for real situations in physics and other fields. For example, driven diffusive lattice gases may model solid electrolytes (Marro *et al* 1991, and references therein), and reaction-diffusion Ising systems are relevant to spin diffusion in magnets, chemically reacting systems and population genetics (see, for instance, Smoller 1983). It has been claimed that *non-equilibrium models* may also be relevant to understanding some of the poorly explained peculiar macroscopic behaviour of certain materials involving microscopic disorder such as spin glasses (Garrido and Marro 1991), magnetically diluted systems (Garrido and Marro 1992) and random-field systems (López-Lacomba and Marro 1992, which we shall refer to as paper I hereafter). The argument behind this claim is that, even though most familiar models of these situations only involve quenched impurities, some of the reported *unusual* observations in real systems might also be related to the possible diffusion of disorder. For example, one may assume that actual impurities diffuse due to a thermally activated random atomic migration, which is an effect contained in the non-equilibrium models. However, the only exact results which issued from the work reported in I mainly concerned one-dimensional systems. In an effort to provide a more convenient description that might allow some contact with experimental observations a *non-equilibrium spin-glass model* has been studied by a kinetic mean-field method in the *pair approximation* (Alonso and Marro 1992).

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Following the same philosophy, we describe here a kinetic mean-field theory in the zeroth- and first-order approximations for the *non-equilibrium random-field model* in I. The latter system has the added interest that it may probably be implemented in the laboratory, as discussed below; we thus hope that the model and its laboratory version may help to understand some of the essential features in disordered systems.

2. Definition and interpretation of the model

Consider a simple (hyper-) cubic d-dimensional lattice, Ω. Let us denote by $\mathbf{s} \equiv \{s_{\mathbf{r}} = \pm 1; \mathbf{r} \in \Omega\}$ any spin configuration which is in contact with a heat bath at temperature T, and by $P(\mathbf{s}; t)$ the probability of $\mathbf{s}$ at time t. The system evolves in time according to the master equation (see, for instance, Glauber 1963, van Kampen 1981):

$$\partial P(\mathbf{s}; t)/\partial t = \sum_{\mathbf{s}^r} [c(\mathbf{s} \mid \mathbf{s}^r) P(\mathbf{s}^r; t) - c(\mathbf{s}^r \mid \mathbf{s}) P(\mathbf{s}; t)]. \tag{2.1}$$

Here, $c(\mathbf{s}^r \mid \mathbf{s})$ represents the rate per unit time for transitions from $\mathbf{s}$ to $\mathbf{s}^r$; the latter differs from $\mathbf{s}$ in that the spin at site $\mathbf{r}$ has been flipped, i.e. $s_{\mathbf{r}} \to -s_{\mathbf{r}}$, as in the so-called *Glauber* (1963) *mechanism*. A main distinguishing feature of the model of interest is however that the kinetics involve the simultaneous competition of several independent (random) Glauber spin-flip mechanisms. That is,

$$c(\mathbf{s}^r \mid \mathbf{s}) = \int_{-\infty}^{+\infty} \mathrm{d}h \, f(h) \, c(\mathbf{s}^r \mid \mathbf{s}; h). \tag{2.2}$$

Here, h represents the applied magnetic field, which is to be interpreted as a random variable with a normalized distribution $f(h)$, and each of the involved elementary Glauber mechanisms has an associated rate denoted by $c(\mathbf{s}^r \mid \mathbf{s}; h)$. As is usual, the latter is assumed for simplicity to satisfy individually a detailed balance condition, i.e.

$$c(\mathbf{s}^r \mid \mathbf{s}; h) = c(\mathbf{s} \mid \mathbf{s}^r; h) \exp[-\beta \Delta H_h] \tag{2.3}$$

where $\Delta H_h \equiv H(\mathbf{s}^r; h) - H(\mathbf{s}; h)$, with respect to some specific Hamiltonian. For the sake of simplicity also, the latter will be taken to be of the Ising type, i.e.

$$H(\mathbf{s}; h) = -J \sum_{\mathrm{NN}} s_{\mathbf{r}} s_{\mathbf{r}'} - h \sum_{\mathbf{r}} s_{\mathbf{r}} \qquad \text{for all } h \tag{2.4}$$

where the first sum is over nearest-neighbour (NN) pairs of sites.

The *simultaneous* competition (2.2) of independent canonical mechanisms makes the system similar to the magnetic Glauber or kinetic Ising model, except that the applied magnetic field changes randomly at each kinetic step according to the distribution $f(h)$. As indicated in I, this has two different interpretations: (a) If one accepts that h represents the field acting on the whole system, as suggested by (2.4), the model corresponds to the case of a magnetic system under the action of a fluctuating magnetic field, or more precisely, under a field which is varying according to $f(h)$ with a period shorter than the mean time between successive transitions modifying the spin configuration. Even though one may guess that this time interval

is relatively short in general, the chances are that such a model situation may indeed by implemented in the laboratory. (Note, however, that it differs essentially from the case (see, for instance, Lo and Pelcovits 1990) in which a system is periodically driven by the action of a field between two ordered phases.) (b) The model admits a different interpretation when one realizes that the elementary Glauber mechanisms are local. That is, given that the elementary rates involve in practice only a local, very small domain of the lattice, the resulting effective rate (2.2) has the same property. Consequently, only the field acting on the spin (at site $\mathbf{r}$) involved by each transition (i.e. $s_r \rightarrow -s_r$) is randomly changed at each kinetic step to have some value h chosen from $f(h)$. Thus, kinetics will soon establish a random *spatial* distribution of local fields, say $f_t^s(h)$, which is a realization of the given $f(h)$, independently of the initial condition $f_0^s(h)$. Consequently, under the latter intepretation, the system may be described (at each time) by the single Hamiltonian

$$H(\mathbf{s};\mathbf{h}) = -J\sum_{\mathrm{NN}} s_r s_{r'} - \sum_r h_r s_r. \tag{2.5}$$

Here, $\mathbf{h} \equiv \{h_r\}$, where h_r is *spatially* distributed according to $f_t^s(h)$. This corresponds to the familiar random-field Ising model (Imry and Ma 1975; see also Imbrie 1986, for instance) except for the fact that $f_t^s(h)$ is continuously changed by the kinetics in such a way that it always maintains itself as a realization of $f(h)$. As discussed with more detail in I, this induces randomness and a sort of (dynamical) frustration having two important features. On the one hand, it essentially differs from the quenched and annealed (equilibrium) random-field cases. On the other hand, the chances are that this kind of frustration may bear some relevance in relation to the macroscopic behaviour of natural disordered systems. It should also be mentioned that the thermodynamics is almost the same for the above two interpretations of the model. An exception concerns the amplitude of the energy fluctuations, which are anomalously large in case (a), given that any field change then affects the whole system, as proved in I; more generally, any macroscopic quantity which is non-linear in h will differ essentially for the two interpretations. Otherwise, the system properties, e.g. phase diagrams that are our main concern here, are the same.

Finally, it may be remarked in order to clarify the nature of the model, that both interpretations have two simple well known limits for $f(h) = \delta(h \pm h_o)$, respectively, where δ is the Dirac delta function and h_o represents a positive constant. Namely, within those two limits, any spin-flip satisfying (2.3) will drive the system to the (unique) equilibrium state corresponding to temperature T and energy $H(\mathbf{s};\mp h_o)$, respectively. For more general distributions $f(h)$, however, the situation is much more involved. In fact, the competition between several field values (equivalently, the random time variations of the spatial distribution of fields) will drive the system asymptotically towards a non-equilibrium steady state in general, as if the spins were acted on by some external non-Hamiltonian agent, whose explicit dependence on $f(h)$, T, J and $c(\mathbf{s}^\tau \mid \mathbf{s})$ is unknown. This is expected to occur for the simplest field distribution describing a crossover between these two limiting conditions, i.e. for

$$f(h) = p\delta(h - h_o) + (1-p)\delta(h + h_o) \tag{2.6}$$

for example. Thus, the model may allow one to analyse a variety of non-equilibrium phase transitions which, as discussed above (cf paper I also, and references therein), might also occur in natural *impure* systems.

The mean-field treatment in the present paper represents a step further within the latter aim, given that the restriction in I to exact results made it necessary to consider only systems fulfilling a certain *global detailed balance* condition which may not be realistic in general. In fact, that condition was only proved to hold in some one-dimensional systems. We shall avoid the use of this or a similar condition in general here. The only restriction below is that, in order to simplify the presentation of results, we shall refer to (2.6) with $p = \frac{1}{2}$ and to rates $c(\mathbf{s} \mid \mathbf{s}^r; h) = \varphi(\beta \Delta H_h)$ in (2.2), where $\beta \equiv (k_B T)^{-1}$ is the inverse temperature, with either

$$\varphi(X) = \min\{1, \exp(-X)\} \tag{2.7a}$$

which corresponds to the algorithm by Metropolis *et al* (1953),

$$\varphi(X) = \{1 + \exp(X)\}^{-1} \tag{2.7b}$$

which is a transition probability introduced by Kawasaki (1972), or

$$\varphi(X) = \exp(-X/2) \tag{2.7c}$$

which has been used by van Beijeren and Schulman (1984) to study the driven diffusive lattice gas. Note that the consideration of several transition rates is interesting given the expected influence of the details of the kinetics on the properties of the steady non-equilibrium state.

3. Zeroth-order mean-field description

The system in section 2 is investigated below by a kinetic mean-field method which corresponds essentially to the first-order theory used before to study, for example, the driven diffusive lattice gas (Garrido *et al* 1990) and the non-equilibrium spin glass (Alonso and Marro 1992). In addition to the first-order approximation which is presented in section 4, we have performed a zeroth-order approximation which is described in the present section. The main motivation for the latter is that it may be compared with the results from a computation by Aharony (1978) of the partition function for the quenched random-field Ising system in a mean-field zeroth-order (equilibrium) approximation. Such a comparison reveals that the two models have some significant differences, even when they are considered in their respective crudest treatment. It also follows that a zeroth-order approach hides some of the non-equilibrium features of the model.

We first remark that the evolution of the magnetization, defined as $m = \langle s_r \rangle$, follows in general from (2.1) as

$$\mathrm{d}m/\mathrm{d}t = F(m) \equiv -2\langle s_r c(\mathbf{s}^r \mid \mathbf{s}) \rangle \tag{3.1}$$

where $\langle\ \rangle$ represents the usual thermal average. Consequently, the homogeneous steady states which are implied asymptotically (for $t \to \infty$) by (3.1) simply correspond to solutions of $F(m) = 0$. On the other hand, our (kinetic) method cannot provide a global (thermodynamic) stability criterion, but only a local stability one: namely, any solution needs to fulfill $(\partial F/\partial m)_{st} < 0$. We also have the possibility of using (3.1) to investigate the trajectories $m(t)$ for different initial conditions, however.

The zeroth-order approximation corresponds to considering a cluster which consists of the spin s_r only, and assuming that the influence of the rest of the system on s_r occurs through a self-consistent mean field. Then,

$$\beta \Delta H_h = 2\beta(qJm + h)s_r \tag{3.2}$$

where q is the lattice coordination number, and consistency simply requires that the mean magnetization around site r, m, equals $\langle s_r \rangle$. Thus, the choices (3.2) and (2.6) with $p = \frac{1}{2}$ lead to $2c(\mathbf{s}^r \mid \mathbf{s}) = \varphi(2\beta[qJm + h_o]s_r) + \varphi(2\beta[qJm - h_o]s_r)$ to be used in (3.1). The properties of the solutions corresponding to the different choices (2.7) may be summarized as follows.

For rates (2.7*c*), the system behaves as for $h = 0$. This result, which may be seen to hold also in the first-order approximation described in section 4, is a consequence of both the even character of $f(h)$ and the peculiar nature of (2.7*c*). Formally, (2.7*c*) admits a factorization that cancels out the contributions from the fields for even distributions, as noted in I. The rate (2.7*c*) is also atypical in the sense that it lacks a proper normalization, and in that it very strongly favours the states of lowest energy; as no reason exists in the present problem to incorporate those effects, we shall restrict ourselves below to the more realistic cases (2.7*a*) and (2.7*b*).

For rates (2.7*b*), the steady states that follow from (3.1) satisfy

$$m = \langle\!\langle \tanh[\beta(qJm + h)] \rangle\!\rangle \tag{3.3}$$

where $\langle\!\langle\ \rangle\!\rangle$ represents the average defined in (2.2). This is precisely the solution obtained by Aharony (1978) after minimizing a free-energy function for the quenched random-field Ising model. Two main differences occur, however. First, the latter corresponds to equilibrium and, consequently, is rate independent. Secondly, we have no free energy which allows us to draw a stable solution such as the one indicated by line 4 in figure 1. In fact, the stable behaviour of the kinetic model in the present approximation needs to follow from (3.1) with $F(m) = \langle\!\langle \tanh[\beta(qJm + h)] \rangle\!\rangle - m$. The first conclusion, which is also evident from (3.3), is that $m = 0$ and $m \neq 0$ are two possible solutions. The former is stable for any $T > T_1(h_o)$ if we define the latter, to be interpreted as a *lower limit of stability*, as the solution of $\partial F/\partial m|_{m=0} = 0$; this implies $qJ\beta_1[1 - \tanh^2(h_o\beta_1)] = 1$, where $\beta_1 \equiv (k_B T_1)^{-1}$. Of course, $m \neq 0$ solutions may coexist with $m = 0$ for $T > T_1$. Consequently, we define an *upper limit of stability*, say $T_2(h_o)$, as the solution of $= \partial F/\partial m|_{m\neq 0} = 0$, where m denotes a solution of $F(m) = 0$; then, $m = 0$ is the only stable solution for $T > T_2$. The lines $T_1(h_o)$ and $T_2(h_o)$ are represented in figure 1. They join together for $T > T^*$ and $h_o < h^*$, where $qJ\beta^* = 3/2$ and $\tanh(\beta^* h^*) = 3^{-1/2}$; $\beta^* \equiv (k_B T^*)^{-1}$. It may be seen that (T^*, h^*) corresponds to a (non-equilibrium) *tricritical* point by developing the RHS of (3.3) in powers of m to write $m = am - bm^3 + \ldots$. That is, the condition $a = 1$ and $b = 0$ marks the onset of a phase transition of second order for $\beta < \beta^*$ and $h_o < h^*$, and the numerical analysis of (3.3) indicates that the phase transition is of first order otherwise. The area enclosed by lines 2 and 3 in the main graph of figure 1 is characterized by the fact that equation (3.1) has two stationary solutions, $m = 0$ and $m \neq 0$, respectively; i.e. $m(t)$ goes asymptotically to one or to the other (for T and h_o given) depending on the initial condition $m(t = 0)$, as in a metastable state. It is remarkable that metastable states may occur for very low values of both T and h_o, as indicated in figure 1.

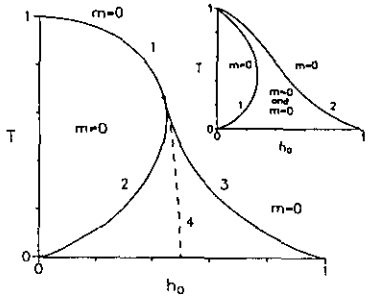

Figure 1. The lower (lines 1 and 2) and upper (lines 1 and 3) limits for local stability of $m = 0$ solutions, $T_1(h_o)$ and $T_2(h_o)$, respectively, for rates (2.7*b*) in the zeroth-order description of section 3. Lines 2 and 3 join at a tricritical point, (T^*, h_o^*). The solution by Aharony (1978) for the quenched system (lines 1 and 4) lies within the area where both $m = 0$ and $m \neq 0$ solutions may occur in the non-equilibrium system. (T is here in units of qJ/k_B; h_o is in units of qJ.) The inset depicts the two limits $T_i(h_o)$, labelled $i = 1$ and 2, respectively, for rates (2.7*a*). The phase transitions are of first order for any T and h_o.

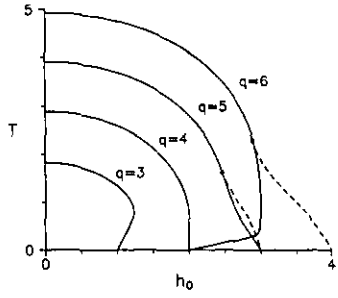

Figure 2. The same as in figure 1 but in the first-order approximation described in section 4 for rates (2.7*b*) and varying values of the lattice coordination number, as indicated. (T is in units of J/k_B; h_o is in units of J.) The indicated tricritical points occur for $(T_1, h_o) = (1.62, 2.46)$ and (2.28, 2.90) when $q = 5$ and 6, respectively. The behaviour for rates (2.7*a*) is qualitatively similar, with the corresponding tricritical points at (1.85, 2.12) and (3.15, 1.95), respectively.

For rates (2.7*a*), the corresponding equations generally present discontinuities for $qJm = h_o$, and it is more convenient to perform the analysis numerically. One exception to this is the analytical result $\exp(2\beta_1 h_o) = 2qJ\beta_1 - 1$. The inset in figure 1 depicts the corresponding behaviour. An outstanding result here is the absence of a critical point, i.e. the phase transition is always of first order, for rates (2.7*a*).

It is interesting to remark that the non-equilibrium system of interest has *in the present approximation* some of the quasi-canonical features found exactly before (López-Lacomba *et al* 1992, and references therein) for $d = 1$. With that aim, one may note that the steady state is characterized by $\partial P(\mathbf{s}; t)/\partial t = 0$ in equation (2.1) and, consequently, by the (sufficient) condition $c(\mathbf{s} \mid \mathbf{s}^r)P_{st}(\mathbf{s}^r) = c(\mathbf{s}^r \mid \mathbf{s})P_{st}(\mathbf{s})$. For the sake of simplicity, we shall restrict ourselves for the rest of this section to a system in which the latter holds; we have no physical justification for it but it provides a *global* stability criterion; in any case, it will not be used for the cases in section 4. It follows that the system may be described then by an *effective free energy* defined by $f = -\beta^{-1} \ln \sum_{\mathbf{s}} P_{st}(\mathbf{s})$. Consequently,

$$f(m) = -\beta^{-1} \ln \sum_{\mathbf{s}} \left[\frac{\langle\langle \varphi(2\beta[qJm + h]) \rangle\rangle}{\langle\langle \varphi(-2\beta[qJm + h]) \rangle\rangle} \right]^{1/2 \sum_{r=1}^{N} s_r} . \tag{3.4}$$

In order to compute (3.4) explicitly, one may write $f = -\beta^{-1} \ln \sum_n g(n) x^{nN/2}$, where $x(m) \equiv \langle\langle c(2\beta\{qJm + h_o\}) \rangle\rangle / \langle\langle c(-2\beta\{qJm + h_o\}) \rangle\rangle$ and $g(n)$ is the number of configurations such that $\sum_r s_r = nN$. For large enough N, this may be approximated by the largest term in the sum, namely, $f \approx \frac{1}{2}(1+\tilde{n}) \ln[\frac{1}{2}(1+\tilde{n})] + \frac{1}{2}(1-\tilde{n}) \ln[\frac{1}{2}(1-\tilde{n})] + \frac{1}{2}\tilde{n} \ln x$ with $\tilde{n} = (1-x)(1+x)^{-1}$. Then, the self-consistency condition, i.e. $\tilde{n} = m$, leads to $\beta f = \frac{1}{2} \ln[\frac{1}{4}(1-m^2)] - \frac{1}{4} m \ln x$. The latter provides a global stability criterion, in particular; e.g. it reveals that the solutions $m \neq 0$

(corresponding to lines 3 and 2, respectively, in the main graph and in the inset of figure 1) are the only stable ones in the non-equilibrium system when the equation $F(m) = 0$ is consistent with $m = 0$ solutions also. This is in contrast to the findings of Aharony for the quenched model (e.g. line 4 in figure 1); moreover, (global) stability depends here on the rate function in general, as indicated by equation (3.4).

4. First-order mean-field description

Let us consider now the more general system in section 2 (i.e. the restrictive condition $c(\boldsymbol{s} \mid \boldsymbol{s}^r)P_{st}(\boldsymbol{s}^r) = c(\boldsymbol{s}^r \mid \boldsymbol{s})P_{st}(\boldsymbol{s})$ is not presumed), and a larger cluster consisting of a spin and its q nearest neighbours. The cluster interacts with the rest of the system via a self-consistent mean field only. Consequently, correlations other than NN ones are not allowed, and $\Delta H_h = 2(nJ + h)s_r$, where $n = -q, -q+2, \ldots, q$ represents the number of spins at NN sites of $\mathbf{r}$ which have state *up* minus those having state *down*. The configuration is then described completely by means of two independent variables, e.g. $m = \langle s_r \rangle$ and $e = \langle s_r s_{r+1} \rangle$. Equivalently, we may use the density of *up* spins, denoted $x = \frac{1}{2}(1+m)$, and the density of *up–up* pairs of NN spins, denoted $z = \frac{1}{4}(1+2m+e)$. It is also convenient to introduce the notation $y \equiv 1-x$, $w \equiv 1+z-2x$ and $v \equiv x-z$, and $\Psi(n) \equiv \langle\langle \varphi(2\beta[2nJ+h]) \rangle\rangle$. Standard techniques (Alonso and Marro 1992) then lead to

$$\frac{\mathrm{d}x}{\mathrm{d}t} = F(x,z;T) \equiv \sum_{n=0}^{q} v^n \binom{q}{n} \left[\frac{w^{q-n}}{y^{q-1}} - \frac{z^{q-n}}{x^{q-1}} \right] \Psi(\tfrac{1}{2}q - n) \qquad (4.1a)$$

and

$$q^{-1}\frac{\mathrm{d}z}{\mathrm{d}t} = G(x,z;T) \equiv \sum_{n=0}^{q} v^n \binom{q}{n} \left[\frac{n w^{q-n}}{y^{q-1}} - \frac{(q-n)z^{q-n}}{x^{q-1}} \right] \Psi(\tfrac{1}{2}q - n). \qquad (4.1b)$$

The steady states may thus be obtained numerically as the solutions of $F(x,z;T) = 0$ and $G(x,z;T) = 0$, and a necessary condition for stability is $(\partial F/\partial x)_{st} < 0$, for instance. The latter may be complemented by integrating numerically (4.1) for several initial conditions.

We have followed in practice a procedure which parallels the one in section 3. That is, one may expect $x = \frac{1}{2}$ ($m = 0$) at high enough T, and one obtains consequently $x = y$, $z = w$ and $F(\frac{1}{2}, z; T) = 0$. The only condition to characterize disordered steady states is therefore

$$G(\tfrac{1}{2}, z; T) = 0. \qquad (4.2)$$

The solution of (4.2) is $z = z(T)$ whose stability requires that $(\partial F/\partial x)_{x=\frac{1}{2},z} < 0$. The breakdown of the latter as T is decreased may be associated with the occurrence of a phase transition. This is of second order when a unique transition (critical) temperature exists, say $T_c \equiv T_1(h_o)$, such that

$$\partial F(\tfrac{1}{2}, z; T)/\partial x|_{x=\frac{1}{2},z} = 0 \qquad (4.3)$$

where $z = z(T)$ is the solution of (4.2). It follows that (4.2)–(4.3) reduce after some algebra to

$$\sum_{n=0}^{q} \binom{q}{n} (\tfrac{1}{2}q - n) \left[\frac{q-2}{q}\right]^n \Psi(\tfrac{1}{2}q - n) = 0 \qquad z = \frac{q}{4(q-1)}. \tag{4.4}$$

This implies, in particular, that $T_c \to 2Jk_B^{-1} \ln[(q-2)q^{-1}]$ as $h_o \to 0$ which corresponds to the Bethe–Peierls equilibrium critical temperature. The phase transition may be of first order, however. Consequently, we define an upper limit of local stability for $m \neq 0$ solutions, say $T_2(h_o)$, as the temperature which makes $(\partial F/\partial x)_{x \neq \frac{1}{2}, z} = 0$, where the values for x and z correspond to the stationary solution of (4.1). Thus, as in section 3, $m = 0$ is the only stable solution for $T > T_2(h_o)$, metastable states (corresponding to locally stable solutions with $m = 0$ and $m \neq 0$) occur for $T_2(h_o) > T > T_1(h_o)$ when both temperatures exist different from each other, and the only globally stable solutions occurring below the *spinodal line* $T_1(h_o)$ are such that $m \neq 0$. The main results from such study may be summarized as follows.

For a one-dimensional lattice, i.e. $q = 2$, equation (4.4) giving T_1 reduces to $z = \frac{1}{2}$ and $\Psi(1) = \langle\langle \varphi(2\beta[2J - h]) \rangle\rangle = 0$. For rates (2.7*b*), this transforms into $(1 + \zeta^2\eta^{-2})^{-1} + (1 + \zeta^{-2}\eta^{-2})^{-1} = 0$, where $\zeta \equiv \exp(-\beta h_o)$ and $\eta \equiv \exp(-2\beta J)$; a solution $T_1(h_o) = 0$ exists only for $h_o < 2J$. That is, the system retains the familiar critical point at zero temperature for low enough fields, while it is overthrown when $h_o > 2J$ in (2.6) with $p = \frac{1}{2}$. For rates (2.7*a*), $\eta^2(\zeta^{-2} + \zeta^2) = 0$ and $1 + \zeta^2\eta^2 = 0$, respectively, for $h_o < 2J$ and $h_o > 2J$. The solution of the former is $T_1(h_o < 2J) = 0$, while there is no real solution for the latter equation. Summing up, the present approximation essentially reproduces the main *exact* results in paper I *when* $d = 1$.

The behaviour for $q > 2$ is summarized in figure 2 depicting $T_1(h_o)$ (solid lines) and $T_2(h_o)$ (dashed lines) when q is varied for rates (2.7*b*); the behaviour is qualitatively similar for rates (2.7*a*) (cf the inset in figure 5 below). In general, the system tends to become macroscopically disordered as h_o is increased, the phase transition is always of second order for $q = 3$ and 4 (e.g. the case of a square lattice), while (non-equilibrium) tricritical points occur for $q > 4$ (e.g. for a three-dimensional simple cubic lattice) which indicate the existence of first-order phase transitions for relatively large values of the field parameter h_o. Note that the metastable region extends over relatively small values of h_o for $q = 6$, but not for $q = 5$ where $T_1(h_o)$ and $T_2(h_o)$ go to zero at the same value of h_o. Figure 3 reveals the existence of metastable states for low temperatures at a given value of the field when $q = 6$. Furthermore, figure 2 indicates that the slope of the $T_1(h_o)$ curve changes sign also for $q = 3$ (but not for $q = 4$). Such behaviour is illustrated with more detail in figure 4; that change of sign, which does not occur for rates (2.7*a*), is the only qualitative difference we have observed between the macroscopic behaviour implied by rates (2.7*a*) and (2.7*b*) in the present approximation.

Finally, we mention that, excluding the variations of the steady state with dynamics we have reported above, most qualitative features of the non-equilibrium system in the present (kinetic) approximation agree with those of the quenched random-field Ising model as revealed by the (equilibrium) first-order mean-field treatments by Bruinsma (1984) and Yokota (1988). A noteworthy exception is the fact that the quenched

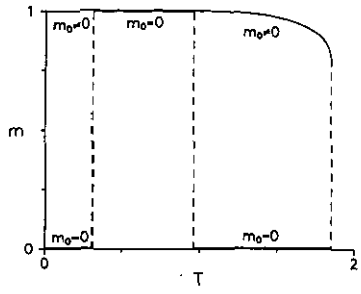

Figure 3. Stationary solutions of equations (4.1) for $q = 6$, rates (2.7*a*), and $h_o/J = 2.75$ when the initial state has magnetization $m_0 = 0$ and $m_0 = 1$, as indicated, to illustrate the existence of metastable states at both low and high (but not at intermediate) temperatures.

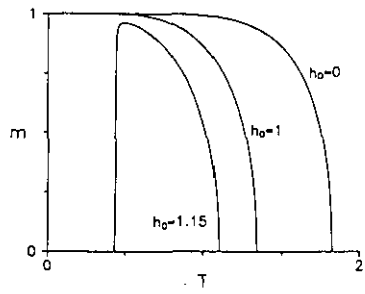

Figure 4. Stationary solutions of equations (4.1) for $q = 3$, rates (2.7*b*) and different values of the field parameter h_o, as indicated, to illustrate the existence of disordered states at low temperatures for some values of h_o.

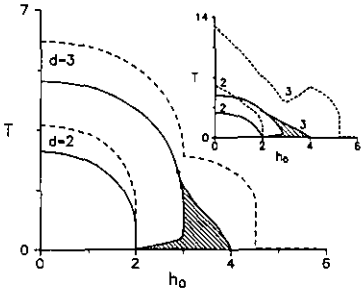

Figure 5. The exact bounds defined in section 5 (dashed lines) compared to the mean-field results in section 4 (solid lines) for rates (2.7*b*) (main graph) and (2.7*a*) (inset) when $d = 2$ and 3, as indicated by the numbers near the curves.

system has a tricritical point for any $q > 3$; i.e. the equilibrium and non-equilibrium models seem to differ essentially in that respect, at least, for $q = 4$.

5. A comparison with exact bounds

Precise bounds defining a region of the phase diagram in which a kinetic lattice system is necessarily ergodic may be found exactly. They are a consequence of a theorem (Ligget 1985) which may be stated roughly as follows: if the effective transition rate is written $c(\mathbf{s}^r \mid \mathbf{s}) = p_o - \sum_\alpha p_\alpha \prod_{r\in\alpha} s_r$, where α represents any set of spins, $p_o = 2^{-N}\sum_r c(\mathbf{s}^r \mid \mathbf{s})$ and $p_\alpha = -2^{-N}\sum_r(\prod_{r\in\alpha} s_r)c(\mathbf{s}^r \mid \mathbf{s})$, the system is ergodic when $\delta \equiv p_o - \sum_\alpha |p_\alpha| > 0$. Consequently, one may use (2.2) and (2.7) to find a relation between T and h_o which makes $\delta = 0$. The explicit expression of that relation for the d-dimensional non-equilibrium random-field model with rates (2.7*b*) and (2.7*c*) has been reported before (López-Lacomba and Marro 1992). The inset in figure 5 depicts the corresponding result for the more familiar (also, analytically more involved) case of rates (2.7*a*). Figure 5 contains also a comparison between the exact bounds for rates (2.7*a*) and (2.7*b*), on the one hand, and the mean-field results in section 4, on the other. The comparisons in figure 5 suggest that the bounds implied by the theorem above are relatively accurate and, consequently, may be useful in practice, especially for the latter case of rates.

6. Conclusions

The present paper deals with a lattice interacting-spin (alternatively, particle) model with competing kinetics whose exact solution was reported before for a one-dimensional lattice (López-Lacomba *et al* 1992, López-Lacomba and Marro 1992). The system time evolution is stochastic due to the competition of two (or more) spin-flip (alternatively, creation–annihilation) mechanisms which involve a random external magnetic field (alternatively, chemical potential) in addition to the usual heat bath. The competing kinetics induce a sort of dynamical frustration which might occur in real disordered systems. In fact, it may be implemented in the laboratory, e.g. by exposing a magnet to a field which is continuously varying according to $f(h)$ (with a period much shorter than the mean time between successive transitions modifying the spin configuration). In general, this will drive the *spin system* asymptotically towards a non-equilibrium steady state (i.e. the competing kinetics acts in practice as an external agent, and the asymptotic state is not an equlibrium state of $\mathbf{s}$ in general), unlike the case for the annealed and quenched random-field models. The differences between the quenched, annealed and non-equilibrium models may be interpreted as follows: while the local field is randomly assigned in space according to a distribution $f(h_r)$ which remains frozen-in for the quenched case, and $f(h_r)$ contains essential correlations in the annealed system, where the impurity distribution is in equilibrium with the spin configuration, the non-equilibrium case in a sense is similar to the quenched system *at each time* during the stationary regime, but h_r keeps randomly changing with time (very fast), also according to $f(h)$, at each site. Consequently, while frustration does not occur in the annealed case, some randomness and frustration influence the behaviour of the non-equilibrium system. These effects are dynamic, however, so that macroscopic differences should be observable between the non-equilibrium and the quenched cases.

We have reported here an analysis of the model for several values of the coordination number, $1 < q \leqslant 6$, by a kinetic mean-field method used before in other non-equilibrium problems (see, for instance, Garrido *et al* 1990, Alonso and Marro 1992). The zeroth-order description, which corresponds to the Bragg–Williams approximation in equilibrium, confirms the existence of some essential differences from the corresponding solution for the quenched system (Aharony 1978). For instance, the steady state strongly depends on the transition rate (2.3) involved by (2.1) in the non-equilibrium system, e.g. a tricritical point separating first- from second-order phase transitions occurs for some choices of transition rates but not for others. On the other hand, the model may be described within the zeroth-order approximation by means of an *effective free energy* (which is rate dependent); this is a feature found exactly before for some one-dimensional cases (López-Lacomba *et al* 1992). Nevertheless, the system for $d > 1$ lacks in general such a (say) *quasi-canonical* feature in a first-order description which corresponds to the Bethe–Peierls approximation in equilibrium. Novel features are then a dependence of the phase diagram on q, the tendency of the system to get disordered at lower minimum values for T and h_o as q is decreased, and the existence of metastable states near $T = 0$ when h_o and q are large enough. This is qualitatively similar to the case of the quenched system in the same approximation (Bruinsma 1984, Yokota 1988), but some interesting differences occur. For example, the non-equilibrium system only has a tricritical point for $q > 4$, while this occurs for $q > 3$ in the quenched case.

The latter fact suggests more dramatic differences (e.g. concerning critical

behaviour) may exist between the non-equilibrium and quenched cases when performing a more realistic description. In any case, the known exact results for $d = 1$ and the present approximate description advises a detailed study of the non-equilibrium system for $d > 1$. In fact, it has motivated us to initiate both a Monte Carlo study and an analytical treatment going beyond the first-order mean-field approximation. It would be very interesting to investigate also some of the practical realizations of our system as described above.

Finally, we remark on a fact concerning the possible experimental realizations of a random-field system. Namely, some studies (Fishman and Aharony 1979; see also, Birgeneau *et al* 1982 and references therein) have described a relation in equilibrium between a quenched random-field model and a diluted antiferromagnet (whose spins are only present at each lattice site with a given probability) in a uniform field; in fact, the latter is considered as a practical realization of the former. Thus, it is interesting to check whether such a relation also holds for non-equilibrium systems in a mean-field approximation. A non-equilibrium dilute antiferromagnet under a constant field may be modelled (Alonso and Marro 1992) by considering competing kinetics which involve a distribution $g(J) = p\delta(J + J_0) + (1 - p)\delta(J)$ of exchange energies (instead of fields), with $J_0 > 0$, and a uniform field h. It follows that those two non-equilibrium systems behave quite distinctly, independently of the order of the approximation investigated. In particular, the non-equilibrium dilute antiferromagnet has $m \neq 0$ solutions at any temperature under the presence of any uniform external field, and no broken symmetry from $m = 0$ to $m \neq 0$ occurs, in contrast to the non-equilibrium random-field system above. Given that a similar result was reached exactly for the one-dimensional non-equilibrium system before (López-Lacomba *et al* 1992), it seems one should conclude that the mentioned relation is an equilibrium feature which holds only for quenched disorder but breaks down if the systems are far from equilibrium.

Acknowledgments

We are indebted to Julio F Fernández and Pedro L Garrido for very useful discussions. This work was supported by Dirección General de Investigación Científica y Técnica, Project PB88-0487, Plan Andaluz de Investigación (Junta de Andalucía), and Comisión de las Comunidades Europeas, C11.0409.

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